Characterizing Modal Spacing of the One-Arm Test Arm During Ring Heater Operation

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1 Motivation

The One-Arm Test (OAT) at LIGO Hanford Observatory (LHO) studies the component systems for monitoring and control of one aLIGO arm cavity, including the Arm Length Stabilization (ALS) system and the Thermal Compensation System (TCS). Cavity scans have been used as part of the studies of the ALS and TCS. This document details the motivations for the cavity scan, describes how cavity scans are performed, and presents preliminary results from cavity scans taken during the operation of the ring heater on the ITMY.

The LIGO gravitational-wave interferometer is a Michaelson interferometer with Fabry-Perot arm cavities. Resonance in these Fabry-Perot cavities increases the effective optical path length in the arm, which increases sensitivity to gravitational waves. The resonance condition of the cavity is controlled with Pound-Drever-Hall (PDH) locking. In PDH locking, phase sidebands are added to the carrier laser so that the sidebands are anti-resonant with the cavity. When the cavity comes out of resonance, the carrier light leaks from the cavity and beats with sidebands. [1] This produces an error signal for use in control servos.

During a cavity scan, a second modulation is applied to the laser frequency. The response of a Fabry-Perot cavity to laser frequency modulation contains information about the cavity parameters, including cavity length, free spectral range, modal spacing, and the radius of curvature of the cavity optics. [2, 3]

The free spectral range (FSR) of a Fabry-Perot cavity is defined as the frequency of a round-trip through the cavity at the speed of light, or

$$\nu_{FSR} = \frac{c}{2L} \tag{1}$$

Fabry-Perot cavities act as filters, admitting only light with frequencies

$$\nu = N \times \nu_{FSR}, \quad N \in \mathbb{N}. \tag{2}$$

Therefore, cavity resonances occur at multiples of the free spectral range. This holds for a zero-order Gaussian beam. In an imperfectly aligned system, the cavity is expected to admit a linear superposition of Hermite-Gaussian spatial modes, TEM_{mn}. These Hermite-Gaussian modes have different resonant frequencies [4], given by:

$$\frac{\nu}{\nu_{FSR}} = \frac{(m+n+1)}{\pi} \arccos(\sqrt{g_1 g_2}). \tag{3}$$

If higher-order Hermite-Gaussian modes are present in the input beam, resonances will also occur at these frequencies. The frequencies of the higher-order modes are dependent on the length of the cavity and the curvature of the cavity optics through the g-parameters [4],

$$g_n \equiv \left(1 - \frac{L}{R_n}\right), \quad n = 1, 2. \tag{4}$$

The product of the g-parameters, $G = g_1 g_2$, is the cavity g-factor. It is important to note that in aLIGO and the OAT, the radius of curvature of the cavity optics is less than the length of the cavity [5], and the g-parameters are negative.

The radius of curvature of an individual mirror is calculated from its g-factor by

$$R_1 = \frac{L}{1 - g_1}. \tag{5}$$

However, in measurements of modal spacing, or the difference between the free spectral range frequency and the resonant frequency of the higher order modes, the g-parameters cannot be separated from the g-factor. Instead, a weighted average radius of curvature is measured:

$$\bar{R}_{curv} = \frac{L}{1 + \sqrt{g_1 g_2}} \tag{6}$$

In addition, the dynamic resonance with the PDH phase modulation sidebands allows for an independent measurement of cavity length and free spectral range. [3] Given a known PDH modulation frequency ν_{PDH}, a dynamic resonance ν_{res} will occur at a frequency such that

$$\nu_{PDH} = (N-1)\nu_{FSR} + \nu_{res}, \quad N \in \mathbb{N} \tag{7}$$

An appropriate integer N is chosen by assuming the measured ν_{FSR} is correct. This N is then used with the known ν_{PDH} and measured dynamic resonance ν_{res} to find a measured ν_{FSR} and cavity length $L = \left(\frac{c}{2\nu_{FSR}}\right)$ accurate to a few parts per million.

The goal of the aLIGO Thermal Compensation System (TCS) is to reduce the effect of thermal noise in the aLIGO optics through uniform heating. Ring heaters are used to apply heat from the outside of the optic, causing thermal expansion. This is expected to decrease the radius of curvature of the test mass, which will in turn increase the cavity g-factor and decrease the modal spacing of the cavity. The goal of cavity scans is to measure cavity characteristics and characterize their time evolution in response to heating of the optics.

2 Performing a Cavity Scan

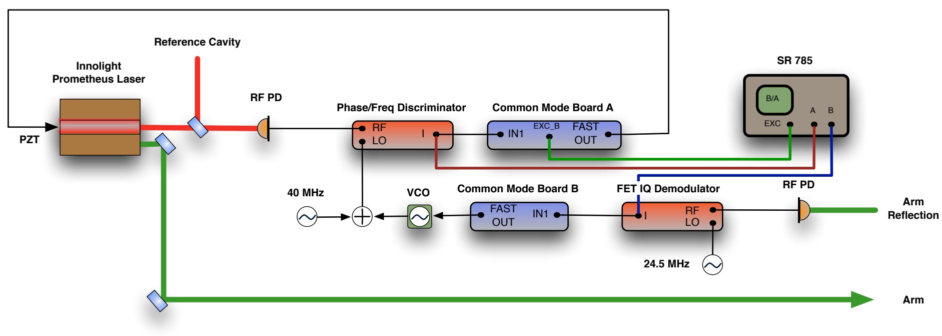

Figure 1: Diagram of arm length stabilization (ALS) system with cavity characterization test points

While the 532 nm ALS laser is locked to the arm cavity, laser frequency modulations are injected into the Innolight Prometheus frequency-doubled laser through the laser frequency servo (Common Mode A) with an SR785 signal analyzer. These frequency modulations are transmitted into the 1064 nm beam used for PSL phase locking and the 532 nm beam used for arm cavity locking. The PSL phase locking beam does not interact with the arm cavity, and the signal from the RF photodiode is used as a proxy for the signal injected into the arm cavity. The arm cavity reflection photodiode gives the output signal, which the SR785 divides by the input signal to produce a transfer function.

Automation of SR785 measurements is necessary to perform and store transfer functions in quick succession over a period of hours. A Prologix GPIB-Ethernet controller is used for remote control and retrieval of data from the SR785. Scripts for performing transfer functions and retrieving data using the GPIB-Ethernet interface were created for use at the 40m interferometer, and were used for cavity scan transfer functions here. The Python script `TFSR785.py` performs a single transfer function.

A bash script, `autoTF`, was used to repeatedly call this Python script. This script was originally set to perform one scan from 30kHz to 80kHz, alternate smaller scans around the first-order modes (from 46-47 kHz and from 65.75-66.75 kHz) three times, then repeat until 12 hours elapses or the script is terminated. During the scan, it was found that the first-order peaks moved further than anticipated, so the script was altered to scan from 45-46 kHz and 66.75-67.75

kHz. In future scans, the smaller scans will be run from 45-47 kHz and 65.75-67.75 kHz for the duration. For all scans, the amplitude of the excitation was 10 mV, and 10 averages were used.

The ALS laser output power is 100 mW, and the SR785 excitation output current is 100 mA. Therefore, the modulation depth for these cavity scans is

$$h = \frac{\sqrt{0.001 \text{ W}}}{\sqrt{0.1 \text{ W}}} = 0.1, \tag{8}$$

giving frequency sidebands with 1% of the carrier power.

3 Data from August 23, 2012

On August 23, 2012, a scan was performed to characterize the effect of heating the ITM using the ring heater, using 630 mA of current over 3.5 hours. The ITM ring heater was turned on at 13:59:30 local time (20:59:30 UTC), and was turned off at 17:31:00 local time (0:31:00 UTC). Throughout this time period, the `autoTF` script was used to run repeated cavity scans. Two such cavity scans, approximately 2.5 hours apart, were analyzed.

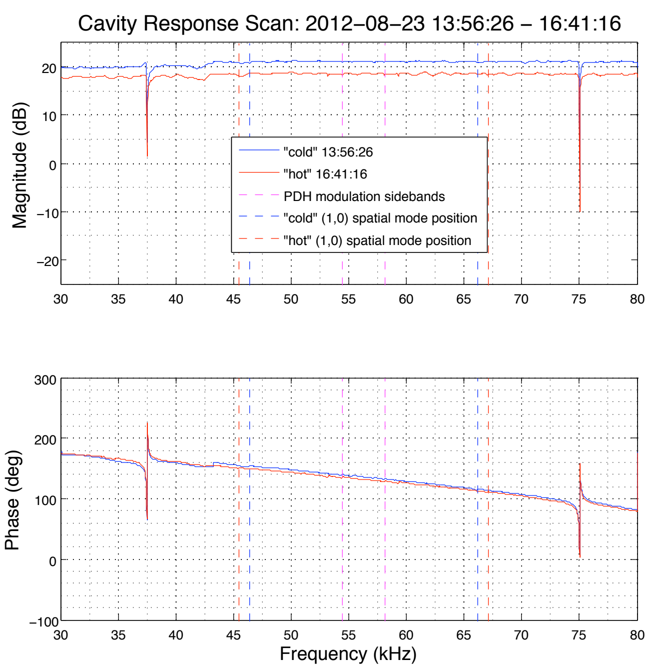

Figure 2: 30kHz - 80 kHz response of cavity

The modal spacing in the cavity decreases from 8.9202 kHz to 7.9268 kHz. This is consistent with the expected decrease in radius of curvature in the ITM. The PDH modulation sidebands remain the same, as expected; the frequency of the phase modulation sidebands is only dependent on the length of the cavity, which is not changed by the ring heater. The change in radius of curvature is calculated from the change in g-factor, assuming that the radius of curvature of the unheated optic remains the same.

$$\begin{aligned}
G &= \left(1 - \frac{L}{R_1}\right)\left(1 - \frac{L}{R_2}\right) \\
\frac{G}{1-\frac{L}{R_0}} &= 1 - \frac{L}{R_2'} \\
1 - \frac{G}{1-\frac{L}{R_0}} &= \frac{L}{R_2'} \\
R_2' &= \frac{L\left(1-\frac{L}{R_0}\right)}{1-\frac{L}{R_0}-G} \qquad (9)
\end{aligned}$$

Further analysis of the transfer functions will be performed to determine the change of the cavity parameters with time.

	initial	heated	change
Modal spacing from first FSR (kHz)	8.9523	7.9021	-1.0502
Modal spacing from second FSR (kHz)	8.8881	7.9515	-0.9366
Mean modal spacing (kHz)	8.9202	7.9268	-0.9934
Cavity length (m)	3996.2	3996.2	0
g-factor	0.5383	0.6203	0.082
Radius of curvature (m)	2305	2165.5	-139.5

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	Radius of Curvature (m)
ITM	2307
ETM	2312
Weighted average	2309.5

References

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[5] Keita Kawabe. H2 one arm test beam geometry. T1200200.