notes

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1 Using HOM Spacing Measurements to Check Various Thermal Actuator Coupling Factors

1.1 Laying out the problem

We have a slurry of known parameters in the IFO's cold state (no Ring Heaters or self-heating), and with some measurements of the Higher Order Mode spacing (HOM) we can constrain or calculate the values of coupling factors of self-heating and ring heater power to surface curvature changes.

Table of physical parameters we know in the **cold** state:

Parameter	Symbol	Value	Unit
ITMY Radius of Curvature	$R_{\text{i, c}}$	1939.2	m
ETMY Radius of Curvature	$R_{\text{e, c}}$	2246.9	m
ARM Length	L	3994.47	m

1.2 Cavity g-factors

We can relate the cavity g-factor (a measure of the test masses' curvature relative to the cavity length) to the HOM spacing data.

Here is a list of symbols used in the derivation:

Parameter	Symbol
ITM g-factor	g_i
ITM g-factor, cold-state	$g_{i,c}$
ETM g-factor	g_e
ETM g-factor, cold-state	$g_{e,c}$
Round-trip Gouy Phase	ψ_{rt}
Curvature, cold-state (ITM, ETM)	$D_{i,c}, D_{e,c}$
Curvature, hot-state (ITM, ETM)	D_i, D_e
Self-heating curvature coupling (ITM, ETM)	A_i, A_e
Ring-heater curvature coupling	B
Self-heating power (ITM, ETM)	$P^i_{\text{self}}, P^e_{\text{self}}$
Ring-heater power (ITM, ETM)	$P^i_{\text{rh}}, P^e_{\text{rh}}$

To start we relate the HOM spacing to the cavity g-factor:

$$
\begin{gathered}
f_{\text{HOM}} = \frac{\psi_{rt}}{2\pi} f_{\text{FSR}} \\
\psi_{rt} = 2 \arccos\left(\sqrt{g_i g_e}\right) \\
g_i g_e = \cos^2\left(\pi \frac{f_{\text{HOM}}}{f_{\text{FSR}}}\right)
\end{gathered}
$$

Then we express the test mass g-factors in terms of the thermal contributions

$$
\begin{gathered}
g_i = 1 - LD_i, \quad D_i = D_{i,c} + A_i P^i_{\text{self}} + BP^i_{\text{rh}} \\
g_e = 1 - LD_e, \quad D_e = D_{e,c} + A_e P^e_{\text{self}} + BP^e_{\text{rh}}
\end{gathered}
$$

Multiplying them together and assuming second order terms are small:

$$
\begin{gathered}
g_i g_e = 1 - L\left(D_{i,c} + D_{e,c} + A_i P^i_{\text{self}} + A_e P^e_{\text{self}} + B(P^i_{\text{rh}} + P^e_{\text{rh}})\right) \\
+L^2\left(D_{i,c}D_{e,c} + D_{i,c}(BP^e_{\text{rh}} + A_e P^e_{\text{self}}) + D_{e,c}A_i P^i_{\text{self}} + \ (A_i A_e + (A_i + A_e)B + B^2 \text{ terms})\right)
\end{gathered}
$$

The terms on the right side are assumed to be small and negligible.

$$
\begin{gathered}
g_i g_e = 1 - L(D_{i,c} + D_{e,c}) + L^2 D_{i,c} D_{e,c} - L\left(A_i P^i_{\text{self}} + A_e P^e_{\text{self}} + BP^e_{\text{rh}}\right) \\
+L^2\left(D_{i,c}(BP^e_{\text{rh}} + A_e P^e_{\text{self}}) + D_{e,c} A_i P^i_{\text{self}}\right)
\end{gathered}
$$

We can gather terms by groups of cold state terms, B depedent terms, and self-heating terms:

$$
\begin{gathered}
g_i g_e = G_c - BLP^e_{\text{rh}}(1 - LD_{i,c}) + X_s = G_c + X_s - BLP^e_{\text{rh}} g_{i,c} \\
\cos^2\left(\pi \frac{f_{\text{HOM}}}{f_{\text{FSR}}}\right) = G_c + X_s - BLP^e_{\text{rh}} g_{i,c} \\
G = G_c + X_s - BLP^e_{\text{rh}} g_{i,c}
\end{gathered}
$$

where,

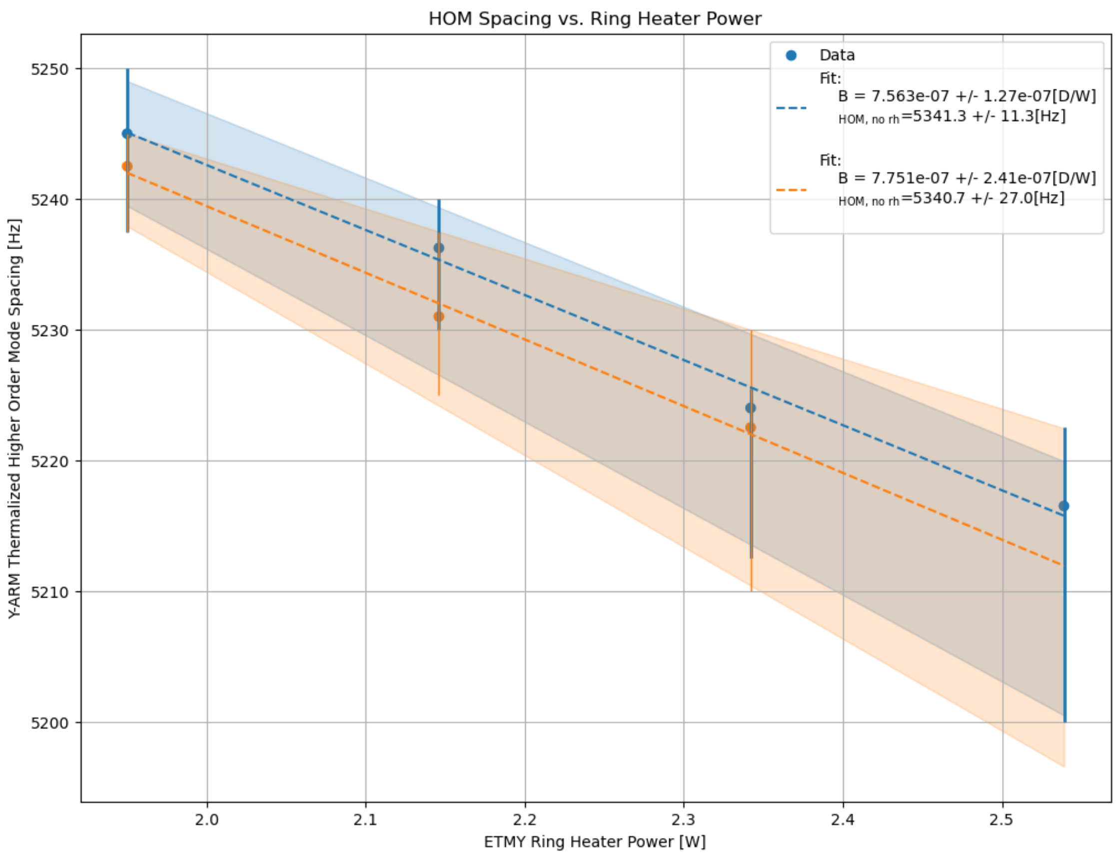
$$
\begin{gathered}
G_c = 1 - L(D_{i,c} + D_{e,c}) + L^2 D_{i,c} D_{e,c} = g_{i,c} g_{e,c} \\
X_s = -L\left(A_i P^i_{\text{self}}(1 - LD_{e,c}) + A_e P^e_{\text{self}}(1 - LD_{i,c})\right) = -L\left(A_i P^i_{\text{self}} g_{e,c} + A_e P^e_{\text{self}} g_{i,c}\right)
\end{gathered}
$$

1.3 Fitting data to recover B and constrain X_s

By fitting a line ($y = mx + b$) to the HOM data as a function of $P^e_{\rm rh}$, we can recover the HOM change due to self heating as well as the coupling of ring-heater power to curvature (B):

$$\cos^2\left(\pi \frac{f_{\rm HOM}}{f_{\rm FSR}}\right) = G_c + X_s - BLP^e_{\rm rh} g_{i,c}$$

$$y = \cos^2\left(\pi \frac{f_{\rm HOM}}{f_{\rm FSR}}\right),\ m = -BLg_{i,c},\ x = P^e_{\rm rh},\ b = G_c + X_s$$



2 Using TCS SIM value to project its estimate of HOM spacing shift due to self heating.

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OMC Data estimate of HOM spacing shift due to self-heating alone:
        174.6 Hz, from 5166.1 Hz --> 5340.7 Hz

TCS SIM estimate of HOM spacing shift due to self-heating alone:
        451.5 Hz, from 5166.1 Hz --> 5617.7 Hz

Difference Factor : TCS_SIM / OMC_DATA = 2.59
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3 Use curvefit and HWS data for ITM absorption estimates to estimate hot-state ARM qs

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Using TCS-SIM ITMY surface defocus from self-heating gain (-3.65e-05 D/W), ETMY
surface defocus is
        3.56e-06 diopters
in order to have the same HOM spacing shift from self-heating as observed.
The ratio of ETMY / ITMY surface defocus from self-heating gain is 0.80

The defocus above is positive…indicating the ITMY surface defocus gain or
absorption is too high.
If we use an ITMY surface defocus from self-heating gain of -1.38e-05 D/W, we
get ETMY surface defocus of:
        -8.98e-07 diopters
And this yields a ETMY surface defocus from self-heating gain of
        -1.11e-05 D/W, and a gain ratio of 0.80
if we assume that the absorption is the same as that of the ITMY.
The corresponding value in TCS-SIM is -2.93e-05 D/W.

Measured HOM spacing in the XARM: 5312.50 Hz
Predicted HOM spacing in the XARM: 5324.86 Hz

With the following assumed parameters:
        Pxarm: 3.852e+05
        Pyarm: 3.852e+05
        alpha_yi: 3.750e-07
        alpha_xi: 4.300e-07
        beta: 0.000e+00
        B: 7.755e-07
        L: 3.994e+03
        gyic: -1.060e+00
        gyec: -7.778e-01
        gxic: -1.059e+00
        gxec: -7.799e-01
        Gxc: 8.257e-01
        Gyc: 8.243e-01
        Pyirh: 0.000e+00
        Pyerh: 2.146e+00
        Pxirh: 8.500e-01
        Pxerh: 1.950e+00
        Gx: 8.149e-01
        Gy: 8.198e-01
        alpha_e: 2.100e-07

Solution for Ai (self-heating to surface defocus gain):
        -2.571e-05 D/W

Calculated mismatch at BS of 0.428% for the following parameters:
        Pxarm: 385159
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```
Pyarm: 385159
alpha_yi: 3e-07
alpha_xi: 6e-07
beta: 0.8
Larm: 3994.47
B: 7.75e-07
itmx_static_curvature: 0.0005153842189352162
etmx_static_curvature: 0.00044559308439533023
itmy_static_curvature: 0.0005156765676567656
etmy_static_curvature: 0.00044505763496372776
itmx_static_lens: -1.2e-05
itmy_static_lens: 1.7e-06
Pyirh: 0
Pyerh: 2.146
Pxirh: 0.8
Pxerh: 1.95
co2y_power: 1.716
co2y_to_sub_dfs: -1.3e-05
itmy_rh_to_sub_dfs: -9e-06
itmy_self_heating_sub_dfs_gain: 0.000487
itmy_thickness: 0.19957
co2x_power: 1.711
co2x_to_sub_dfs: -1.1e-05
itmx_rh_to_sub_dfs: -9e-06
itmx_self_heating_sub_dfs_gain: 0.000487
itmx_thickness: 0.19957
Ai: -2.33e-05
alpha_e: 1.0344e-07
n: 1.45
bs_thickness: 0.0604
bs_aoi: 0.7853981633974483
ITMY_to_BS: 5.3328
ITMX_to_BS: 5.3828
```